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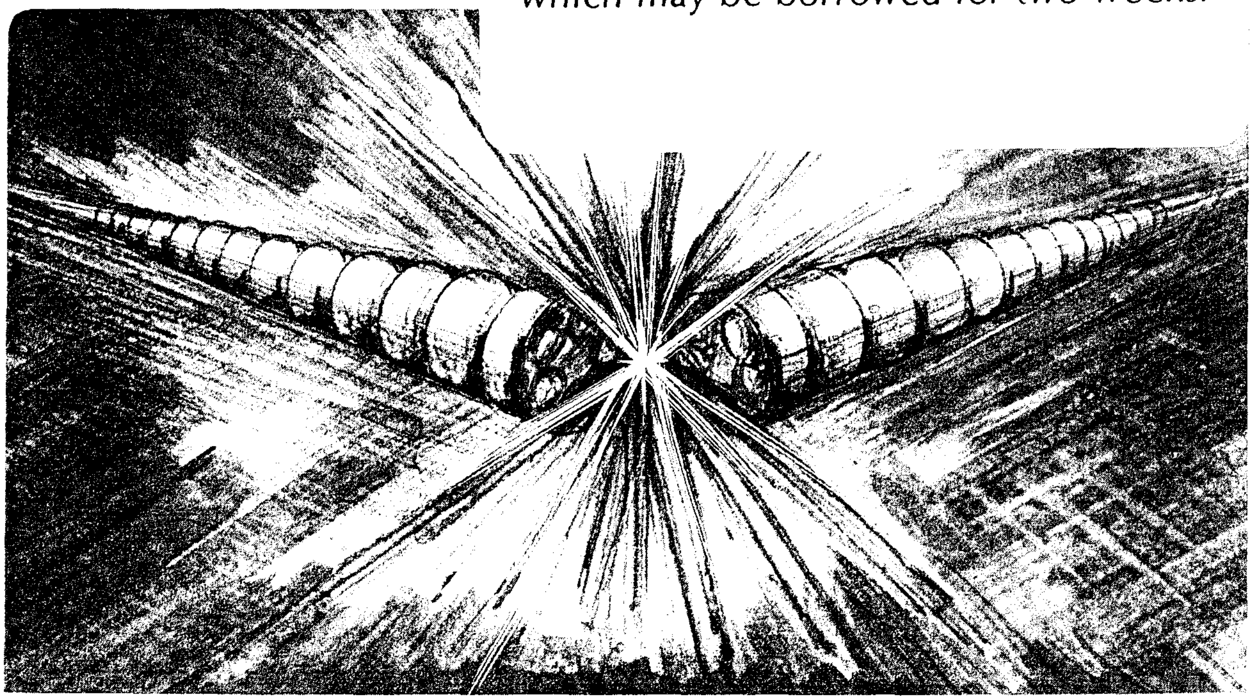

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PHASE AND AMPLITUDE CONTROL OF THE RADIO FREQUENCY WAVE IN THE TWO-BEAM ACCELERATOR*

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ABSTRACT

The sensitivity of the radio frequency (rf) wave generated by the free electron laser portion of a Two-Beam Accelerator (TBA) is analyzed, both analytically and numerically in a "resonant particle" approximation. It is shown that the phase of the rf wave is strongly dependent upon errors in the wiggler strength and wavelength and upon the electron beam characteristics of energy and current. The resulting phase error is shown to be unacceptable for a TBA, given reasonable errors in various components. A feedback system is proposed which will keep the rf wave phase within acceptable bounds. However, the feedback system is, at best, cumbersome and a simpler system would be desirable.

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I. INTRODUCTION

The Two-Beam Accelerator (TBA) concept was first introduced a few years ago and, subsequently, described in a number of review papers [1]. More detailed treatments can be found in three recent articles [2,3,4].

Basically the idea is simply to have a low-energy beam travel through an undulator magnet and, hence, by the free electron laser (FEL) mechanism generate microwave radiation. The energy of the low-energy beam is repeatedly resupplied by induction units and the microwave radiation is employed to accelerate, to very high energies, the desired particles ("the high-energy beam"). The device is proposed for future linear colliders where high gradients (and hence reduced overall length and, consequently, reduced capital cost) and high power efficiency (and hence reduced operating cost) are important considerations. A schematic is shown in Fig. 1 and a set of possible parameters (taken from Ref. [3]) is given in Table I.

In order to operate well (i.e., to produce high energy particles with a well-defined, and repeatable, energy) the TBA must incorporate, in its FEL portion, tight control of the rf wave amplitude and phase. An identification of this need, and a rough bounding of the magnitude of the effect, was given in previous papers [5,6].

It is the point of this paper to present a comprehensive treatment of the subject; details of which can be found in an unpublished note [7]. In Sec. II we formulate the problem using a resonant particle approximation. Analytic solutions are obtained, for some special cases, in Sec. III. A complete numerical analysis is given in Sec. IV. The numerical work, and analytic work, shows, clearly, that without feedback the TBA will not be an acceptable device. Thus, the feedback system becomes an essential part of the TBA. Finally, in Sec. V an adequate feedback system is proposed.

II. FORMULATION OF THE PROBLEM

We employ a "resonant particle" approximation to the FEL in order to study rf phase and amplitude sensitivity to the various parameters which characterize the FEL. Employing the standard notation of Kroll, Morton, and Rosenbluth [8] we may describe the FEL by the following equations:

$$\frac{d\gamma}{dz} = -\frac{\omega a_w a_s \sin\psi}{c\gamma} + \frac{2\alpha_o \omega^2 a_{so}^2}{\omega_{po}^2} , \tag{1}$$

$$\frac{d\psi}{dz} = (k_w - \delta k_s) - \frac{\omega}{2c\gamma^2}(1 + a_w^2 - 2a_s a_w \cos\psi) + \frac{d\phi}{dz} , \tag{2}$$

$$\frac{da_s}{dz} = \frac{\omega_p^2 a_w \sin\psi}{2\omega c\gamma} - \alpha\, a_s , \tag{3}$$

$$\frac{d\phi}{dz} = \frac{\omega_p^2 a_w \cos\psi}{2\omega c a_s \gamma} . \tag{4}$$

In these equations, γ is the energy of the resonant particle, ψ is the phase of this particle with respect to the rf wave, a_s is the normalized intensity of the rf wave, and ϕ is the phase of the rf wave. The normalized undulator amplitude, the normalized rf field, and the beam plasma frequency, ω_p, are given by:

$$a_w = 0.093\ \lambda_w(\mathrm{cm})\ B_w(\mathrm{kG}) \quad , \tag{5}$$

$$a_s = 6.05 \times 10^{-6}\ \lambda(\mathrm{cm}) \sqrt{\frac{P(w)}{a(\mathrm{cm})b(\mathrm{cm})/2}} \quad , \tag{6}$$

$$\omega_p^2 = \frac{6.6 \times 10^{20}\ I(\mathrm{kA})}{a(\mathrm{cm})b(\mathrm{cm})/2} \quad , \tag{7}$$

where the FEL wave guide dimensions are a x b. The quantity δk_s is:

$$\delta k_s = \left(k_s^2 + \frac{\pi^2 m^2}{a^2} + \frac{\pi^2 n^2}{b^2}\right)^{1/2} - k_s \ , \tag{8}$$

where ω is the frequency of the rf and k_s, the wave number of the rf, is ω/c.

The energy taken out of the FEL is represented by the parameter α and the α-term in the first equation represents the replenishing of the FEL electron's energy by the induction units. In this model, the energy gain and loss is continuous; i.e., the discrete, and periodic, nature of the TBA is neglected.

We study, in the next two sections, the sensitivity of the rf wave (i.e., a_s and φ) to variation in a_w, α, ω_p, and the initial conditions on γ, ψ, and a_s.

III. ANALYTIC SOLUTION

In order to gain some insight into the FEL behavior, a linear analysis was made. This approximation is valid, since only small perturbations are considered.

In Eq. (2), we substitute the expression for $d\phi/dz$ from Eq. (4), so that only the first three equations have to be solved simultaneously. We define some abbreviations for the right hand side of Eqs. (1 to 4) to give compact notation, where p represents the various parameters.

$$\frac{d\gamma}{dz} \equiv \Gamma(\gamma,\psi,a_s,p) \equiv \Gamma,\ \frac{d\psi}{dz} \equiv \Psi,\ \frac{da_s}{dz} \equiv A,\ \frac{d\phi}{dz} \equiv \Phi, \tag{9}$$

We define the incremental variables, where subscript 0 represents equilibrium values:

$$\gamma_1 \equiv \gamma - \gamma_0\ ,\ \psi_1 \equiv \psi - \psi_0\ ,\ a_{s1} \equiv a_s - a_{s0},\ p_1 \equiv p - p_0.$$

The linearized equations are:

$$\frac{d\gamma_1}{dz} = \gamma_1 \frac{\partial\Gamma}{\partial\gamma} + \psi_1 \frac{\partial\Gamma}{\partial\psi} + a_{s1} \frac{\partial\Gamma}{\partial a_s} + p_1 \frac{\partial\Gamma}{\partial p}\ ,$$

$$\frac{d\psi_1}{dz} = \gamma_1 \frac{d\Psi}{d\gamma} + \psi_1 \frac{d\Psi}{d\psi}\ , \text{ etc.}$$

where $\frac{\partial\Gamma}{\partial\gamma} = \frac{\partial\Gamma}{\partial\gamma}(\gamma_0, \psi_0, a_{s0}, p_0)$, etc.

We define the derivatives in compact notation:

$$\frac{\partial\Gamma}{\partial\gamma} \equiv \Gamma_\gamma\ ,\quad \frac{\partial\Gamma}{\partial\psi} \equiv \Gamma_\psi\ ,\quad \frac{\partial\Gamma}{\partial a_s} \equiv \Gamma_a\ ,\quad \frac{\partial\Gamma}{\partial p} \equiv \Gamma_p\ ,$$

$$\frac{\partial\psi}{\partial\gamma} \equiv \Psi_\gamma\ , \text{ etc.,}\quad \frac{\partial A}{\partial\gamma} \equiv A_\gamma\ , \text{ etc.}$$

The three linearized equations in compact notation, with some rearranging, are:

$$\frac{d\gamma_1}{dz} - \gamma_1 \Gamma_\gamma - \psi_1 \Gamma_\psi - a_{s1} \Gamma_a = p_1 \Gamma_p\ , \tag{10}$$

$$\frac{d\psi_1}{dz} - \gamma_1 \Psi_\gamma - \psi_1 \Psi_\psi - a_{s1} \Psi_a = p_1 \Psi_p\ , \tag{11}$$

$$\frac{d_{as}}{dz} - \gamma_1 A_\gamma - \psi_1 A_\psi - a_{s1} A_a = p_1 A_p\ . \tag{12}$$

We assume a general solution of the homogeneous equations to be:

$$\gamma_1 = B_\gamma e^{\omega_1 z} + C_\gamma e^{\omega_2 z} + D_\gamma e^{\omega_3 z} \quad ,$$

$$\psi_1 = B_\psi e^{\omega_1 z} + C_\psi e^{\omega_2 z} + D_\psi e^{\omega_3 z} \quad ,$$

$$a_{s1} = B_a e^{\omega_1 z} + C_a e^{\omega_1 z} + D_a e^{\omega_1 z} \quad .$$

By substituting the solution into Eqs. (10 to 12), and equating ω_1 terms, we get:

$$\omega_1 B_\gamma - B_\gamma \Gamma_\gamma - B_\psi \Gamma_\psi - B_a \Gamma_a = 0 \quad ,$$

$$\omega_1 B_\psi - B_\gamma \Psi_\gamma - B_\psi \Psi_\psi - B_a \Psi_a = 0 \quad ,$$

$$\omega_1 B_a - B_\gamma A_\gamma - B_\psi A_\psi - B_a A_a = 0 \quad .$$

We set the determinant equal to zero, as the necessary condition for a non-trivial solution; (where the B's are the unknowns) and drop the subscript on ω, since all three sets of ω terms give the same result:

$$\begin{vmatrix} \omega - \Gamma_\gamma & -\Gamma_\psi & -\Gamma_a \\ -\Psi_\gamma & \omega - \Psi_\psi & -\Psi_a \\ -A_\gamma & -A_\psi & \omega - A_a \end{vmatrix} = 0 \ .$$

This gives the cubic equation for ω:

$$\omega^3 - \omega^2 (\Gamma_\gamma + \Psi_\psi + A_a)$$

$$+ \omega (\Gamma_\gamma \Psi_\psi + \Psi_\psi A_a + A_a \Gamma_\gamma - \Gamma_a A_\gamma - \Psi_a A_\psi - \Gamma_\psi \Psi_\gamma)$$

$$- \Gamma_\gamma \Psi_\psi A_a - \Gamma_\psi \Psi_a A_\gamma - \Gamma_a \Psi_\gamma A_\psi + \Gamma_a \Psi_\psi A_\gamma$$

$$+ \Gamma_\gamma \Psi_a A_\psi + \Gamma_\psi \Psi_\gamma A_a = 0 \ . \tag{13}$$

An approximate analytical solution can be obtained for zero current ($\omega_p^2 = 0$) where many terms drop out, and the roots are:

$$\omega_1 = 0 \quad , \tag{14}$$

$$\omega_2,\ \omega_3 = \pm\ (\Gamma_\psi \psi_\gamma)^{1/2} \quad . \tag{15}$$

These two roots are pure imaginary, giving undamped oscillations as expected.

The inhomogeneous equations (10-12) will have solutions of the form:

$$\gamma_1 = \Gamma_I\ p_1 + \text{(homogeneous solution)} \quad , \tag{16}$$

$$\psi_1 = \Psi_I\ p_1 + \text{(homogeneous solution)} \quad , \tag{17}$$

$$a_{s1} = A_I\ p_1 + \text{(homogeneous solution)} \ . \tag{18}$$

When these solutions are substituted back into equations (10-12), terms of the same form as the p_1 terms on the right will be obtained. We make the substitutions, ignoring the homogeneous solution terms, as they average to zero, and divide out p_1, as it appears in all terms. We obtain:

$$- \Gamma_I\ \Gamma_\gamma \quad - \Psi_I\ \Gamma_\psi \quad - A_I\ \Gamma_a \quad = \quad \Gamma_p \quad , \tag{19}$$

$$- \Gamma_I\ \Psi_\gamma \quad - \Psi_I\ \Psi_\psi \quad - A_I\ \Psi_a \quad = \quad \Psi_p \quad , \tag{20}$$

$$- \Gamma_I\ A_\gamma \quad - \Psi_I\ A_\psi \quad - A_I\ A_a \quad = \quad A_p \quad . \tag{21}$$

This can be expressed in matrix notation and numerically solved for the I-subscript terms on an HP-15C calculator or by other methods, when a specific parameter, p, is chosen, so that the p-subscript terms can be evaluated.

The phase, ϕ, is a constantly increasing quantity. We are interested in the ϕ deviation, $\Delta\phi$, the departure from the ϕ value at the equilibrium condition. We define some abbreviations involving ϕ:

$$\frac{d\phi}{dz} \equiv \Phi\ (\gamma,\ \psi,\ a_s,\ p) \equiv \Phi \quad , \tag{22}$$

$$\frac{\partial\Phi}{\partial\gamma}\ (\gamma_0,\ \psi_0,\ a_{s_0}\ ,\ p_0) \equiv \Phi_\gamma \quad , \quad \frac{d\Phi}{d\psi} = \Phi_\psi \quad , \text{ etc.}, \tag{23}$$

where the derivatives are evaluated at the equilibrium condition.

Equation (4) for $\frac{d\phi}{dz}$ can be linearized

$$\frac{d\phi}{dz} = \Phi + \Phi_\gamma\ \gamma_1 + \Phi_\psi\ \psi_1 + \Phi_a\ a_{s1} + \Phi_p\ p_1 \quad . \tag{24}$$

We integrate with respect to z, omitting the equilibrium term, to get:

$$\Delta\phi = (\Phi_\gamma\ \gamma_1 + \Phi_\psi\ \psi_1 + \Phi_a\ a_{s1} + \Phi_p\ p_1)z$$

In order to determine the continued growth of $\Delta\phi$, independent of the oscillatory terms, we need the inhomogeneous solution terms only. From Eqs. (16-18) for γ_1, ψ_1, and a_{s1}:

$$\Delta\phi = (\Phi_\gamma\ \Gamma_I + \Phi_\psi\ \Psi_I + \Phi_a\ A_I + \Phi_p)\ p_1\ z \quad .$$

The Φ terms can be obtained from Eqs. (23-24), and using $p_1 = \Delta\omega_p{}^2$

$$\Delta\phi = \Phi_0 \left(-\frac{\Gamma_I}{\gamma_0} - \frac{\Psi_I \sin\psi_0}{\cos\psi_0} - \frac{A_I}{a_{s0}} + \frac{1}{\omega_{p0}^2} \right) \Delta\omega_p{}^2\ z \tag{25}$$

$$\text{where } \Phi_o = \frac{\omega_{po}^2\ a_{wo} \cos\psi_o}{2\omega c\ a_{s0}\ \gamma_0} \quad . \tag{26}$$

The last two terms in Eq. (25) are numerically the most significant, and of opposite sign. A simplified analytical solution can be obtained, in order to determine the possibility of reducing $\Delta\phi$ by cancellation.

We write the determinant solution for A_I, with $p = \Delta\omega_{p0}^2$, from Eqs. (19-21)

$$A_I = \frac{\begin{vmatrix} \Gamma_\gamma & \Gamma_\psi & -\Gamma_{\omega_p^2} \\ \Psi_\gamma & \Psi_\psi & -\Psi_{\omega_p^2} \\ A_\gamma & A_\psi & -A_{\omega_p^2} \end{vmatrix}}{\begin{vmatrix} \Gamma_\gamma & \Gamma_\psi & \Gamma_a \\ \Psi_\gamma & \Psi_\psi & \Psi_a \\ A_\gamma & A_\psi & A_a \end{vmatrix}} \tag{27}$$

In order to determine what terms are important, numbers are inserted and numerical values are used to decide which terms to drop, in order to simplify the results. After some manipulation, the result is

$$A_I \approx \frac{a_{s0}}{2\omega_{p0}^2} \quad .$$

The last two terms in the bracket in Eq. (25), which are the significant ones, are

$$-\frac{A_I}{a_{s0}} + \frac{1}{\omega_{p0}^2} = -\frac{a_{s0}}{2a_{s0}\omega_{p0}^2} + \frac{1}{\omega_{p0}^2} = \frac{-1}{2\omega_{p0}^2} + \frac{1}{\omega_{p0}^2} \quad .$$

The first term is always half the second, so no cancellation is possible.

The complete approximate expression for $\Delta\phi$ is then

$$\Delta\phi \approx \Phi_0 \quad \frac{1}{2\omega_{p0}^2} \quad \Delta\omega_{p0}^2 \, z \quad ,$$

$$\approx \frac{\omega_{p0}^2 \, a_{w0} \cos \psi_0 \quad \Delta\omega_{p0}^2 \, z}{4\omega c \; a_{s0} \; \gamma_0 \; \omega_{p0}^2} \quad ,$$

$$\approx \frac{a_{w0} \cos \psi_0 \; \Delta\omega_{p0}^2 \; z}{4\omega c \; a_{s0} \; \gamma_0} \quad . \tag{28}$$

This can be recognized, from the last term of Eqs. (22) and (4), as one-half the first order value that would be obtained if only $\Delta\omega_p^2$ was changed. The neglected terms make a noticeable, but not a major, modification to these approximate results.

IV. NUMERICAL RESULTS

Since the analytical solution turned out to be somewhat complicated, we have written a program for the IBM PC that numerically integrates the four coupled differential equations by the second order Runge-Kutta method.

The program accepts errors in ω_p, a_w, k_w, α, and energy addition by induction units, of arbitrary duration and amplitude. It also accepts initial non-equilibrium values for the dynamic variables γ, ψ, and a_s. We have taken, for this study, the parameters given in Ref. 3, as shown in Table I.

The important output is the ϕ deviation, $\Delta\phi$, which is the error in phase of the rf delivered to the electron linac. This translates to energy spread of the high energy beam in a non-linear fashion.

Figure 2 shows ϕ deviation plotted against distance along the FEL. Errors in ω_p, k_w, α, and the energy addition by induction units gives an approximately linearly increasing phase deviation for a little more than the duration of the error, with an approximately constant phase deviation thereafter. The phase deviation is linear with error amplitude and duration, and multiple sources of error added linearly.

However, an error in a_w gives a different result, as shown on Fig. 3. The ϕ deviation increases rapidly for about 15 m and then increases slowly for the duration of the error. Then the ϕ deviation decreases rapidly to a low value.

Table II gives the allowable amplitude of the errors for a 0.05 radian phase deviation for 30 and 100 m error durations. For definiteness, 0.05 radian (3°) of ϕ deviation is used as an acceptable error.

Figure 4 shows the effect of initial value errors with γ_0 error plotted as an example. With initial non-equilibrium values for the dynamic variables γ, ψ, and a_s, the ϕ deviation continually increases, for the full duration of the run.

Table III gives the allowable initial non-equilibrium errors for a phase deviation of 0.05 radians for several different error accumulation lengths. Error accumulation length is used instead of FEL section length in the expectation that error correction stations can be placed periodically along an FEL section.

In Ref. 4, estimates of $\Delta\phi = 0.14$ radian and $\Delta a_s/a_{s0} = 2.1\%$ were made for parameter errors of 0.1% and L = 100 m, without consideration of differential coupling between variables. Table IV shows the values given by the program. The estimates in Ref. 4 for $\Delta\phi$ were within a factor of two for the important parameter changes. The estimate in Ref. 4 for $\Delta a_s/a_s$ error was quite pessimistic. Also, the $\Delta a_s/a_s$ errors did not continually increase with distance as the uncoupled expressions would predict, but returned to very low values after an initial increase.

V. FEEDBACK CONTROL BY USING THE FIELD AMPLITUDE

Regular feedback using error detection, amplification, and application of a correcting signal is not workable. Amplifier delay will be on the order of 15 to 20 ns. This means that the error detected from the head of the beam and the rf pulse cannot cause a correcting signal until 5 to 6 meters of the beam has passed. Furthermore, the perturbation of a quantity from equilibrium gives a continually increasing $\Delta\phi$, as shown in Fig. 4. As a result, overcorrection and instability would result.

It is therefore necessary to devise a correction scheme which is automatic and essentially instantaneous. A method which appears workable is to change the amplitude of a_s as a function of $\Delta\phi$, by the addition of a phase stable reference rf to the FEL rf, at a number of correction stations spaced along the FEL. The addition would be by directional couplers, as shown on Fig. 5. The reference rf is added at 90° phase difference with respect to the FEL rf, as shown on Fig. 6a. As the FEL rf changes phase by $\Delta\phi$, the reference rf then has a component in phase with the FEL rf which changes the amplitude of a_s as a function of $\Delta\phi$, as shown in Fig. 6b.

Directional couplers couple both directions, so power will be coupled out of the FEL into the reference rf line. There will be a small change in amplitude and phase of the FEL rf as a result of the coupling, which can be corrected by adjusting the FEL waveguide parameters slightly.

The power extracted from the FEL by the directional coupler will go into the reference rf feed line. Directional coupler characteristics are such that if the coupled reference rf feeds into the FEL in the forward direction, the coupled FEL power feeds into the reference rf line also in the forward direction. In order not to contaminate the necessarily highly phase stable reference rf, 320 MW blocks of rf will be split off by septum couplers and terminated at the correcting station.

A change in a_s from equilibrium, once imposed to correct an error in $\Delta\phi$, will give continued $\Delta\phi$ accumulation, similar to the γ_0 error in Fig. 4. Detecting this $\Delta\phi$ accumulation at subsequent correction stations, and making corrections just gives a $\Delta\phi$ error that oscillates from plus to minus along the FEL, with an amplitude approximately equal to the original error.

In order to get a true error reduction, it is necessary to turn off the $\Delta\phi$ accumulation by applying an equal and opposite correcting signal at the end of the FEL section. This is implemented using the bypass waveguide arrangement shown on Fig. 5, and with more detail on Fig. 7. The FEL rf is split into two nearly equal parts, with the larger (to allow for attenuation) going through the bypass waveguide and the other going through the main FEL. This latter goes through a short amplification section, without power take-off, to bring the power back up to 5 GW. There will be some phase error,

$\Delta\phi_1$, generated in going through the first section of the FEL. This is mixed with the error-free power from the bypass waveguide to get an error of $\Delta\phi_1/2$ for the combination. A correction in a_s is then made at the beginning of the next FEL section, sufficient to give a change of $-\Delta\phi_1$ in going through the FEL section, leaving an error of $-\Delta\phi_1/2$ at the end. An equal and opposite correction is then made at the end in order to eliminate the $\Delta\phi$ accumulation. The bypass waveguide rf with $+\Delta\phi_1/2$ error is then mixed with the FEL rf with $-\Delta\phi_1/2$ error, giving zero net error. Thus an error developed in one section is fully corrected in the next section with no residual error accumulation remaining.

For $\Delta\phi_1/2 = 0.025$ radian, $\Delta a_s/a_{s0} = 0.0004$ is needed to get the required correction in 100 m. The added inphase reference rf is given by

$$\frac{\Delta a_s}{a_{s0}} = \frac{C \sin \Delta\phi}{a_{s0}}$$

where $C^2 \sim$ reference rf power

$$\frac{C}{a_{s0}} = \frac{\Delta a_s}{a_{s0} \sin \Delta\phi} = \frac{0.0004}{\sin 0.025} = 0.016 \quad .$$

The power ratio of reference rf to FEL rf is then $(0.016)^2 = 2.56 \times 10^{-4}$. The reference rf $= 2.56 \times 10^{-4} \times 5 \times 10^9$ W $= 1.28$ MW. With a 24 db coupler, 320 MW is required in the reference line.

The reference rf has to be phase stable to a few degrees over 2 km, which implies a frequency stability of $\approx 10^{-8}$. In order to measure the frequency this accurately, from a practical standpoint, the clock reference has to be cw. The reference rf power is too high to be cw, so a pulsed FEL amplifier is used to go from a reasonable cw level (say 100 kW from a gyrotron amplifier) up to 15 GW. 4.5 GW will be used to drive the main FEL, 5 GW will be used as the reference rf, and 5.5 GW will go through the bypass waveguide. This FEL amplifier has to be phase stable to a few degrees during the pulse, but pulse-to-pulse phase stability is not important, since its output feeds both the main FEL and the reference rf.

The reference rf will flow in a waveguide parallel to the FEL, with 320 MW extracted at each correction station with a septum coupler. Since 40 correcting stations require 12.8 GW total, and there will be about 10 db attenuation in an oversize waveguide, about 7 FEL boost amplifiers of about 2 db gain each will be needed in the reference rf line. For such a small gain, these amplifiers should have adequate phase stability.

Temperature control will be needed on the bypass waveguide and the FEL waveguide to keep phase change due to length change within allowable limits.

VI. CONCLUSION

The analysis using the "resonant particle" approximation shows that phase errors will be unacceptable for a TBA without some feedback system for phase correction. The proposed feedback system appears workable, although it requires a substantial amount of extra equipment and is somewhat complicated. It is expected that an analysis using many particles would give similar results.

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Table I Parameters for a 1 TeV x 1 TeV Two-beam Accelerator Collider

Low Energy Beam	
Average beam energy (units of mc^2)	40
Beam current	2.15 kA
Bunch length	6 m
Wiggler wavelength	27 cm
Average peak wiggler field	2.4 kG
Beam power	43 GW
Beam energy	0.8 kJ
Power production	2.2 GW/m
Number of FEL injectors	2 x 2
Power from mains	160 MW
High Gradient Structure	
Wavelength	1 cm
Gradient	500 MeV/m
Stored energy	40 J/m
Fill time	18 ns
High Energy Beam	
Injection energy	2 GeV
Repetition rate(f)	0.5 kHz
Final energy	1 TeV
Length	2 x 2 km
Luminosity	4×10^{32} cm^{-2} sec^{-1}
Beam height (σ_W)	0.14 μ
Beam width (σ_Z)	1.4 μ
Single beam power	8.0 MW
Number of particles	10^{11}
Disruption (D)	1.3
Beamstrahlung (δ)	0.2
Overall efficiency (from mains to HEB)	10%

Table II Allowable Errors (Percent) for Different Error Durations for 0.05 Radian Phase Deviation

Type of Error	Duration of Error	
	30 m	100 m
$\Delta\omega_\beta^2/\omega_\beta^2$	0.2	0.06
$\Delta a_w/a_w$	0.3	0.3
$\Delta k_w/k_w$	0.3	0.1
$\Delta\alpha/\alpha$	0.2	0.07
Induction Unit	-0.2	-0.07

Table III Allowable Initial Non-equilibrium Errors (Percent) for 0.05 Radians Phase Deviation

Type of Error	Error Accumulation Length	
	30 m	100 m
$\Delta\gamma_0/\gamma_0$	0.2	0.04
$\Delta\psi_0/\psi_0$	5	1.6
$\Delta a_{s0}/a_{s0}$	0.14	0.04

Table IV Errors in the Quantities Listed, of 0.1% for 100 Meters, Resulted in the Given Values of $\Delta\phi$ and $\Delta a_s/a_s$

	$\Delta\phi$(rad)	$\Delta a_s/a_s$(%)
$\Delta\omega_\beta^2/\omega_\beta^2$	0.09	0.1 max
$\Delta a_w/a_w$	0.02	0.2 max
$\Delta\gamma_0/\gamma_0$	0.12	0.2 max
$\Delta a_{s0}/a_{s0}$	0.13	-
Estimate from Ref. [6]	0.14	2.1

FIGURE CAPTIONS

Figure 1. Conceptual configuration of the Two-Beam Accelerator (TBA).

Figure 2. Values of the deviation in phase, from its nominal value, as a function of distance down the TBA, for errors in the beam plasma frequency. Two different types of errors are considered.

Figure 3. The same phase error, as in Fig. 2, but here for an error in the wiggler a_w.

Figure 4. Phase error, as in Fig. 2, for an initial error in the particle beam energy. Note how the phase error simply increases without bound as a function of distance down the TBA.

Figure 5. Schematic of the proposed feedback system on rf field amplitude.

Figure 6. Phasor diagram of the addition of the FEL rf and the reference rf to get a change in a_s.

Figure 7. Detail of the phase error correction system.

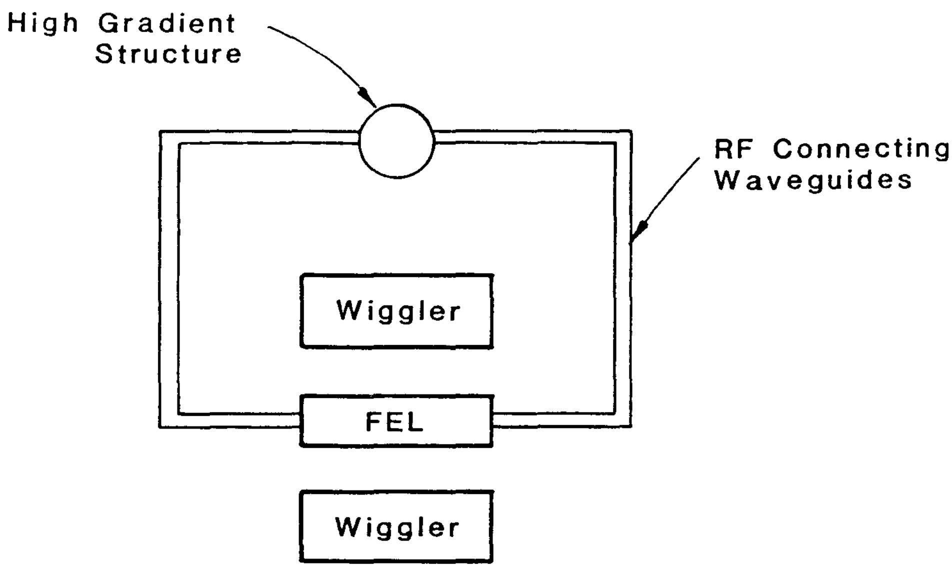


Figure 1a

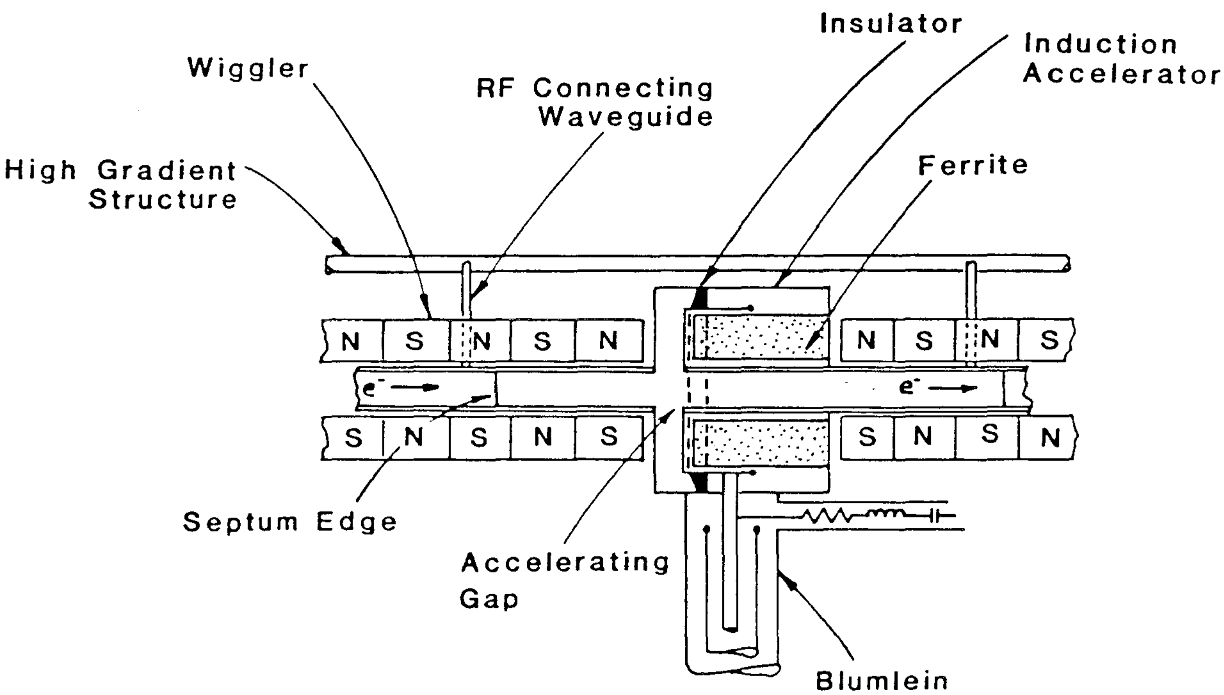
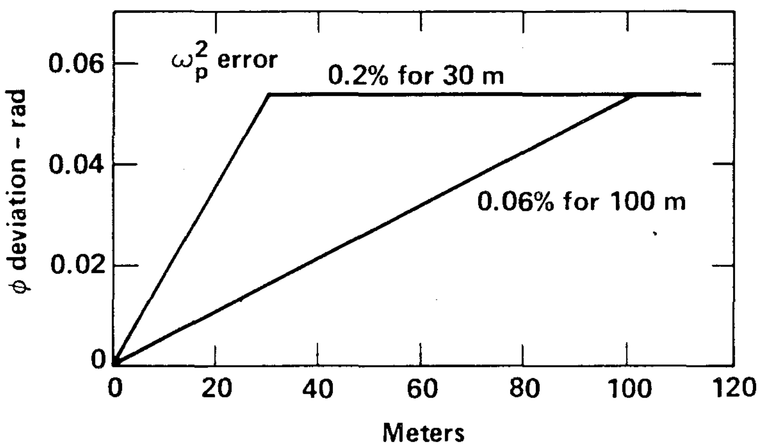
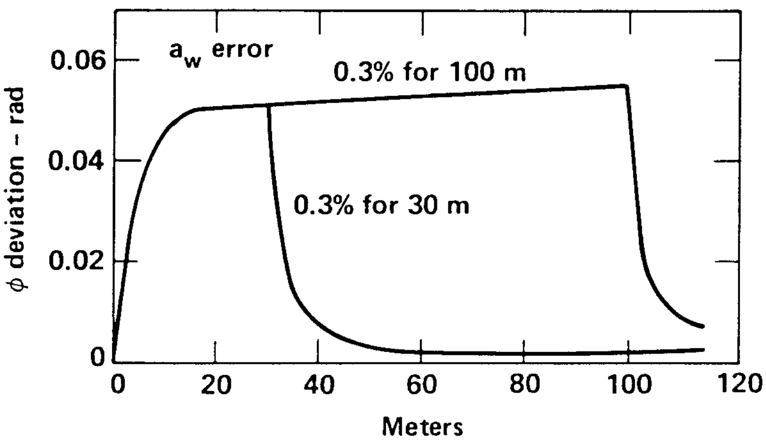


Figure 1b



ω_p^2 error
0.2% for 30 m
0.06% for 100 m
0.06
0.04
0.02
0
ϕ deviation - rad
0
20
40
60
80
100
120
Meters

Figure 2



a_w error
0.3% for 100 m
0.3% for 30 m
ϕ deviation - rad
0.06
0.04
0.02
0
0
20
40
60
80
100
120
Meters

Figure 3

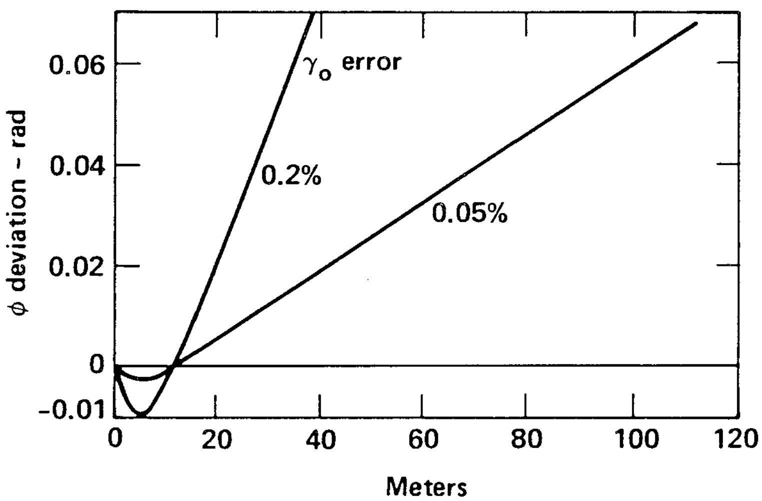

0.06
0.04
0.02
0
-0.01
ϕ deviation - rad
0
20
40
60
80
100
120
Meters
γ_o error
0.2%
0.05%

Figure 4

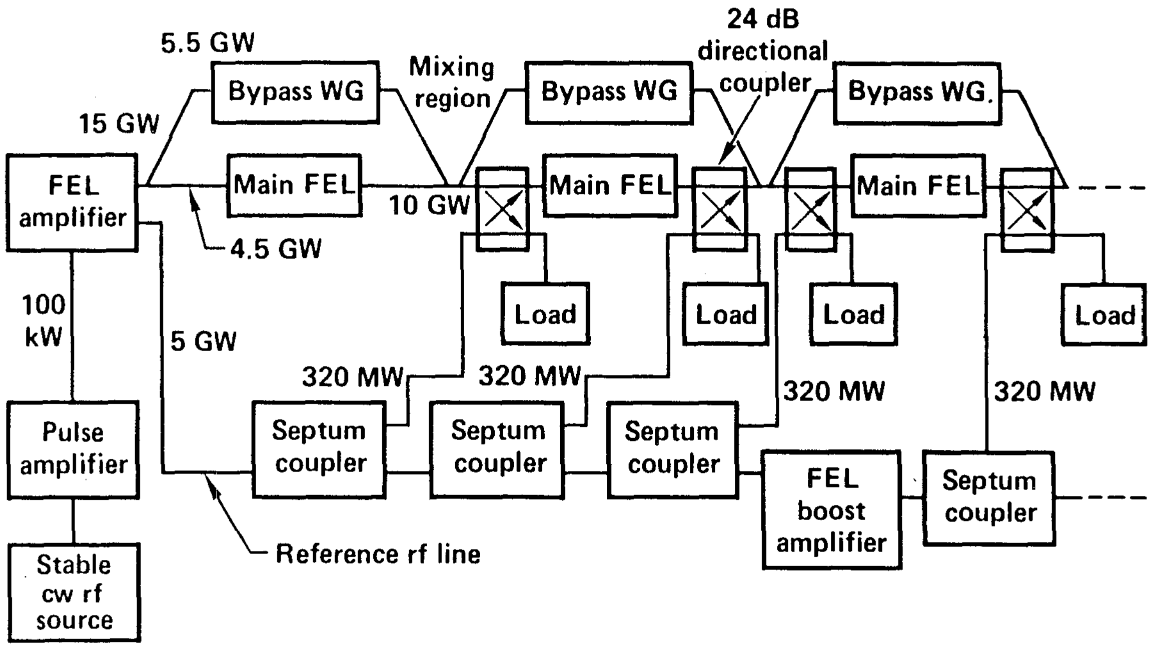

24 dB
directional
coupler
5.5 GW
Bypass WG
Mixing
region
Bypass WG
Bypass WG.
15 GW
FEL
amplifier
Main FEL
Main FEL
Main FEL
10 GW
4.5 GW
Load
Load
Load
Load
100
kW
5 GW
320 MW
320 MW
320 MW
320 MW
Pulse
amplifier
Septum
coupler
Septum
coupler
Septum
coupler
FEL
boost
amplifier
Septum
coupler
Stable
cw rf
source
Reference rf line

Figure 5

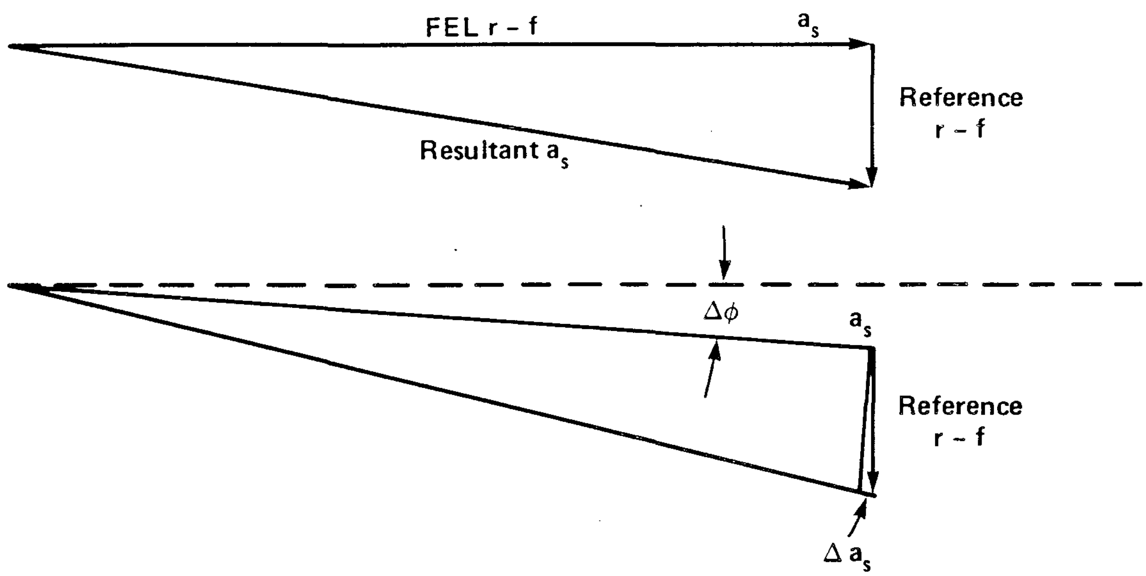

FEL r - f
a_s
Reference
r - f
Resultant a_s
$\Delta\phi$
a_s
Reference
r - f
Δa_s

Figure 6a, 6b

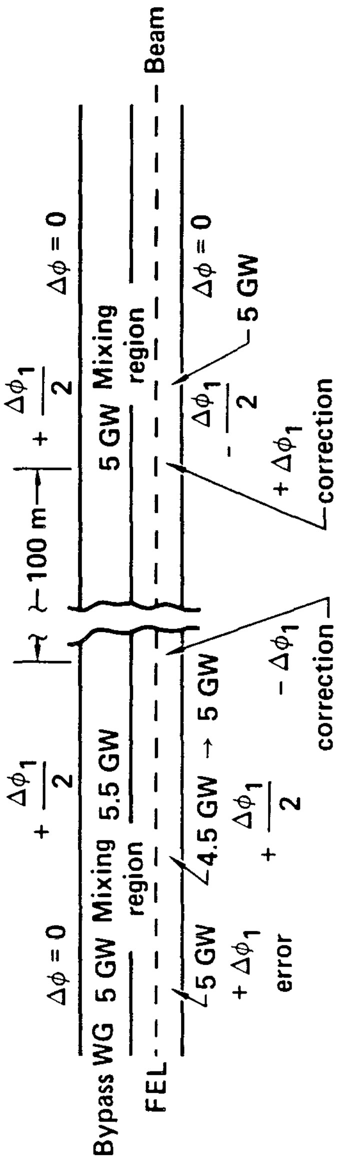

Bypass WG
FEL
Beam
Δφ = 0
5 GW
Mixing region
5.5 GW
4.5 GW → 5 GW
+ Δφ1 error
~100 m
- Δφ1 correction
+ Δφ1 correction
Δφ = 0
5 GW

Figure 7